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Complex analysis of the long jump taking into consideration atmospheric conditions

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Abstract
It is well known that wind and altitude can affect the performance of long-jumpers. In this paper we show that according to our model the maximal allowable tail-wind 2m/s at sea level will improve the distance of a jump of the same athlete by 14-18 cm. Altitude also will improve the performance on a still day by 4-5 cm for each 1000 m. It can be also calculated that a 5 °C increase of temperature is equivalent to an increase of altitude by 150 m. It appears that wind and altitude have a significant effect mainly because of changes in the take-off values. The faster the athlete runs, the greater the take-off velocity. We also provide a long jump computer simulator for modeling the trajectory of the sportsman's center of gravity. Our program is supposed to help coaches and sportsmen estimate the contribution of atmospheric effects into the final result and take full advantage of the sportsman's potential.

Keywords: Long jump, take-off, wind, altitude

1. Introduction

According to IAAF (International Association of Athletics Federations) regulations, sprint and jump performances for which the measured wind-speed exceeds +2.0 m/s are deemed illegal and cannot be ratified for record purposes (IAAF 1998). The average wind component parallel to the track is measured near the jumping pit during an interval encompassing the run-up and the jump. If this measurement exceeds 2 m/s, no record-breaking jump is recognized. Similarly, performances which are achieved at altitudes exceeding 1000 meters above sea level are noted as "altitude-assisted", but unlike their wind-aided counterparts, these can and have qualified for record status [2].

Indeed, the 1968 Olympics saw amazing World Records set in the men's 100 m, 200 m, and Long Jump, thanks in part to the lofty 2250-meter elevation of Mexico City. In making his record jump, Beamon enjoyed a number of advantageous environmental factors. Mexico City's air had less resistance than air would have at sea level. This allows runners to run faster and jumpers to jump farther. In addition to Beamon's record, world records were broken in most of the sprinting and jumping events at the 1968 Olympic Games. Beamon also benefited from a trailing wind of 2 meters per second on his jump, the maximum allowable for record purposes. It has been estimated that the trail wind and altitude may have improved Beamon's long jump distance by 31 cm (12.2 inches) [5]. During the same hour Lee Evans set the world record for 400 meters that lasted for almost 20 years.

Undoubtedly, wind and altitude affect the performance of sprinters and long-jumpers. The International Athletics Union acknowledges this by imposing a special rule relating to wind during long-jump performances.

In this paper we investigate how the atmospheric factors such as wind, altitude and temperature will affect the sportsman during long jump.

2. Materials and methods

2.1 Long Jump Model

To facilitate the study of long jumps, it has been proposed to split the total distance jumped into partial distances, and then to identify the determining factors for each. For the long jump, Linthorn *et al.* [1] classifies the following partial distances as shown in Figure 1.

L_0: Take-off distance: the horizontal distance between the anterior edge of the take-off board and the vertical projection of the center of gravity (CG) at the instant of take-off.

L_1: Flight distance: the horizontal distance covered by the CG while the athlete is free in the air.

L_2: Landing distance: the horizontal distance between the vertical projection of the center of gravity at the instant the heels touch the sand and the mark from where the jump will be measured.

The distance L_1 represents more than 85% of the total distance of a jump and thus has the highest relationship with the final result. We can say that L_1, and thus performance in the horizontal jumping events, is determined by the same four factors affecting movement of all projectiles: take-off height, angle and velocity, and air resistance.

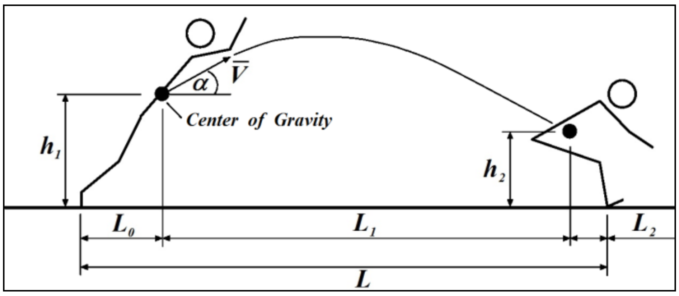

Fig 1: Diagram of a long jump showing contributions to the official distance.

2.2 Long Jump Modeling without Wind

In the absence of wind, the velocity of the jumper relative to the ground is the same as the velocity relative to the air. The long-jumper is modeled as a projectile acted on by constant gravity plus the two components of the total aerodynamic force (drag and lift), which at jumpers' speeds are usually assumed to be proportional to the square of the air speed. With the origin at the position of the jumper's center of mass at take-off, the *x* direction chosen as parallel to the run-up track and the *y* direction chosen as vertically upwards, the governing equation of motion is [4]

$$m\frac{d^2\bar r}{d\,t^2} = -mg\hat k - \frac{1}{2}\rho S C_D |\bar v|^2\,\hat\tau + \frac{1}{2}\rho S C_L |\bar v|^2\,\hat n \qquad (1)$$

Here ρ denotes the density of the air, *S* is a typical cross-sectional area of the jumper, *m* denotes the jumper's mass, $\bar r$ denotes the position vector of the jumper at any time t during the jump, $\bar v$ denotes the corresponding velocity vector, while C_D and C_L denote the drag and lift coefficients respectively. The unit vectors $\hat k$, $\hat\tau$ and $\hat n$ are respectively in the directions vertically upwards, parallel to the jumper's velocity, and perpendicular to the jumper's velocity but lying in the vertical plane through the athlete's center of mass.

At sea level, air density ρ is about 1.226 kg/m^3; air density at 3,000 meters is about 0.905 kg/m^3. Since air density changes in a roughly linear fashion with altitude, we can use the formula ρ = 1.226 - Altitude x [(1.226 - 0.905)/3.000].

Typical values of S range from 0.4 to 0.7 square meters; larger jumpers and less aerodynamic positions result in higher values. Precise calculation is not possible, but you can obtain an estimate by assuming that a jumper weighing 50 kg. will have a frontal area of 0.4 m^2 and that his frontal area increases by 0.0033 square meters with each pound of body weight. So, the sportsman's frontal area, S, will equal approximately 0.0033·W, where W is his body weight.

In this paper the usual assumption is made that SC_D and SC_L are each some average constant for the duration of each long jump. The value for SC_D has been estimated as 0.36 for a sportsman weighing 70 kg from measured values on sprinters, cyclists and speed-skaters quoted in Ward-Smith [5]. To obtain some idea of the effect of lift, a representative value 0.04 is chosen for SC_L.

2.3 Long Jump Modeling with Wind

When a wind $\bar w$ is blowing, the air speed of any projectile is given by

$$\bar v^* = \bar v - \bar w \qquad (2)$$

The drag and lift effects will depend on $\bar v^*$, and only on $\bar v$ in the absence of wind. Therefore, with the addition of wind, the basic equation (1) for the projectile part of the long-jumper's motion becomes [4]

$$m\frac{d^2\bar r}{d\,t^2} = -mg\hat k - \frac{1}{2}\rho S C_D |\bar v^*|^2\,\hat\tau^* + \frac{1}{2}\rho S C_L |\bar v^*|^2\,\hat n^* \qquad (3)$$

where $\hat\tau^*$ is a unit vector in the direction of $\bar v^*$ and $\hat n^*$ is a unit vector perpendicular to $\hat\tau^*$ and lying in the vertical plane.

It seems take-off velocity is the most important factor affecting L_1 and it has a very high relationship with the velocity at the touchdown of the take-off foot at take-off, which in turn is dependent on the approach velocity. In other words, the faster the athlete runs, the greater the horizontal velocity at the instant he/she touches the take-off board and the greater the take-off velocity.

2.4 Take-off Velocity Calculation

Reference [3] gives a simple formula which can be used to correct 100-meter sprint times according to both wind and altitude conditions,

$$t_{0,0} \cong t_{w,H}\cdot[1.03-0.03\cdot\exp(-0.000125\cdot H)\cdot(1-w\cdot t_{w,H}/100)^2], \qquad (4)$$

where $t_{w,H}$ (s) is the recorded race time run with wind w (m/s) and at altitude H (m), while the time $t_{0,0}$ is the adjusted time as if it were run at sea level in calm conditions.

This formula allows us to make a rough approximation of how the take-off velocity of the athlete changes under the influence of wind and altitude. We suppose that the take-off velocity $V_{w,H}$ is proportional to the run-off velocity $V_{run\text{-}off}(w,H)$ of the athlete, where the latter can be approximated by the formula $V_{run\text{-}off}(w,H) \cong V_{sprint}(w,H)=100$ meter/$t_{w,H}$. Then the sportsman's take-off speed corresponding to wind w and altitude H can be estimated using the following formula:

$$V_{w,H} \cong V_{0,0}\cdot(t_{0,0}/t_{w,H}) \qquad (5)$$

Using formulas (2)-(5) we can calculate how the result of a long jump depends on wind and altitude. Our long jump distance calculator requires the following input parameters describing the sportsman's long jump at sea level in still conditions: take-off speed $V_{0,0}$, take-off angle α, take-off height h_1, landing height h_2 and 100-meter race time $t_{0,0}$. Then we can calculate the distance of the jump L, its duration t and the highest position of the sportsman's center of gravity for the selected values of wind speed w, altitude H and temperature T. The values of main parameters for elite male and female athletes have been taken from reliable sources [1, 3] and are shown in Table 1.

Table 1: Values of main parameters.

#	Parameter Name	Parameter value	
		Males	Females
1	Take-off speed $V_{0,0}$ (m/s)	9.46	8.35
2	Take-off angle α (degree)	22	24
3	Sportsman's mass m (kg)	80	64
4	Take-off height h_1 (m)	1.3	1.2
5	Take-off distance L_0 (m)	0.5	0.4
6	Landing height h_2 (m)	0.65	0.6
7	Landing distance L_2 (m)	0.4	0.35
8	100-meter sprint time $t_{0,0}$ (s)	10	11
9	Drag coefficient C_D	0.6599	0.6599
10	Lift coefficient C_L	0.0733	0.0733

3. Results & Discussion

3.1 Calculations in the still air at sea level

First of all, we calculated how the range of a jump launched at α depends on the starting angle α and the starting velocity of the jump V_0. V_0 was between 6 m/s and 10 m/s, while α was changing from 15° to 25°. Other parameters: m, h_1, h_2, L_0 and L_2 have been taken from Table 1 as they are given for male athletes.

The results of calculations are shown in Fig. 2, 3. We see that the increase of α from 15° to 25° increases the distance by 0.7-2.0 m depending on the value of V_0 (Fig. 2). The increase of V_0 from 6 m/s to 10 m/s increases the distance by 2.6-5.0 m in an almost linear manner depending on the value of α (Fig. 3).

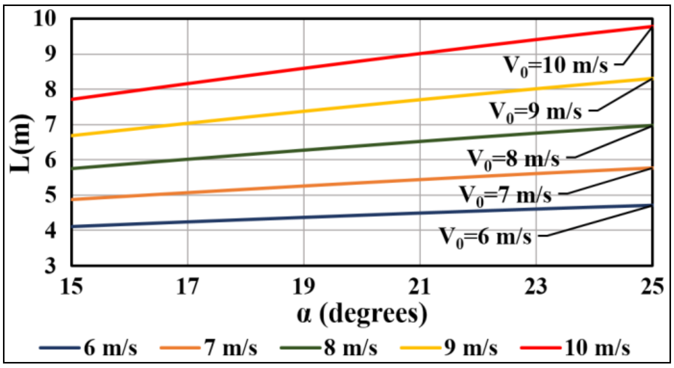

Fig 2: The dependencies of L on the starting angle α for different values of V_0.

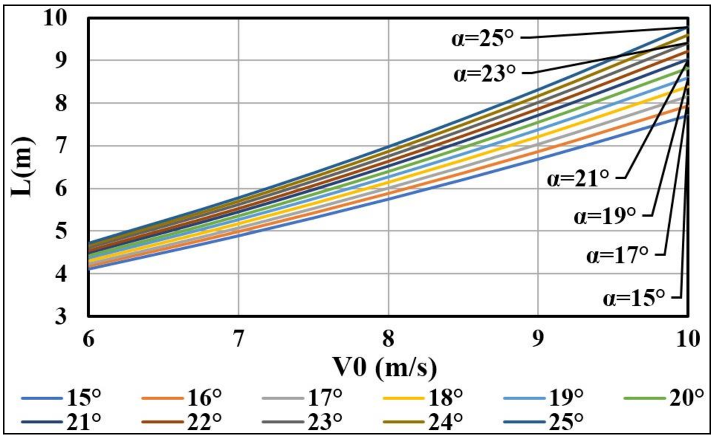

Fig 3: The dependencies of L on the starting velocity V_0 for different values of α.

As you can see, there are a lot of variables that play a role in generating the furthest long jump possible. However, it seems that horizontal velocity is the most important factor in determining overall distance because it directly affects one's flight trajectory and total distance of the jump.

In general, the result of a long jump may be described by the following formula

$$L(V_0, \alpha, m, h_1, h_2, L_0, L_2) = L(V_0, \alpha, m^*, h_1^*, h_2^*, L_0^*, L_2^*) + \Delta L_0 + \Delta h \cdot ctg(\alpha) + \Delta L_2, \qquad (6)$$

where $L^*=L(V_0, \alpha, m^*, h_1^*, h_2^*, L_0^*, L_2^*)$ is the distance corresponding to the elite male athlete (m^*, h_1^*, h_2^*, L_0^* and L_2^* have been taken from Table 1), jumping with the starting parameters V_0 and α, $\Delta h \cdot ctg(\alpha)=(h_1- h_1^*+h_2- h_2^*)\cdot ctg(\alpha)$ - is the change of L caused by changes in the take-off height h_1 and landing height h_2 comparing to their values from Table 1 (we assume that during the landing the sportsman's center of gravity moves along a straight line), $\Delta L_0=L_0-L_0^*$ and $\Delta L_2=L_2-L_2^*$ are the corrections for L_0 and L_2.

3.1 Calculations considering wind and altitude

Formulas (2)-(5) from the previous section allow us to calculate long jump results for both males and females and evaluate how the atmospheric conditions affect the final result. The values of main parameters for elite male and female athletes have been taken from Table 1.

Using formula (4) we calculated values of coefficients ($t_{0,0}/t_{w,H}$) for different $t_{0,0}$, H and w (see Table 2). These coefficients show how a 100-meter sprint time of the sportsman changes under the influence of wind w and altitude H. Coefficients have been calculated for four values of $t_{0,0}$ - 10 s, 11 s, 12 s and 13 s, altitude H takes six values - 0 m, 500 m, 1000 m, 1500 m, 2000 m and 2500 m, and wind is represented by five values - -2 m/s, -1 m/s, 0 m/s, 1 m/s and 2m/s. For $t_{0,0}$, w and H falling between these values we can use linear approximation to calculate ($t_{0,0}/t_{w,H}$).

Table 2: Values of coefficients ($t_{0,0}/t_{w,H}$) for different $t_{0,0}$, H and w.

$t_{0,0}$ (sec)	Wind w(m/s)	Altitude H(m)						
		0	500	1000	1500	2000	2500	3000
10	-2 m/s	0.9866	0.9893	0.9918	0.9941	0.9963	0.9984	1.0003
	-1 m/s	0.9937	0.9959	0.9979	0.9999	1.0017	1.0035	1.0051
	0 m/s	1.0	1.0018	1.0035	1.0051	1.0066	1.008	1.0094
	1 m/s	1.0057	1.0071	1.0085	1.0098	1.011	1.0122	1.0132
	2 m/s	1.0107	1.0118	1.0129	1.014	1.0149	1.0158	1.0167
11	-2 m/s	0.9851	0.9879	0.9905	0.9929	0.9952	0.9973	0.9993
	-1 m/s	0.993	0.9952	0.9974	0.9993	1.0012	1.003	1.0046
	0 m/s	1.0	1.0018	1.0035	1.0051	1.0066	1.008	1.0094
	1 m/s	1.0062	1.0076	1.009	1.0102	1.0114	1.0126	1.0136
	2 m/s	1.0116	1.0127	1.0138	1.0147	1.0157	1.0165	1.0173
12	-2 m/s	0.9836	0.9864	0.9891	0.9916	0.994	0.9962	0.9983
	-1 m/s	0.9923	0.9946	0.9968	0.9988	1.0007	1.0025	1.0042
	0 m/s	1.0	1.0018	1.0035	1.0051	1.0066	1.008	1.0094
	1 m/s	1.0067	1.0081	1.0094	1.0107	1.0118	1.0129	1.014
	2 m/s	1.0125	1.0136	1.0146	1.0155	1.0164	1.0172	1.0179
13	-2 m/s	0.982	0.985	0.9878	0.9904	0.9928	0.9951	0.9972
	-1 m/s	0.9916	0.994	0.9962	0.9982	1.0002	1.002	1.0037
	0 m/s	1.0	1.0018	1.0035	1.0051	1.0066	1.008	1.0094
	1 m/s	1.0072	1.0086	1.0099	1.0111	1.0122	1.0133	1.0143
	2 m/s	1.0134	1.0144	1.0153	1.0162	1.017	1.0178	1.0186

In Tables 3 and 4, you can see jumping length corrections calculated for male (Table 3) and female (Table 4) elite athletes for different altitudes and different wind speeds.

Table 3: Jumping length corrections for men (m).

Wind w(m/s)	Altitude H(m)						
	0	500	1000	1500	2000	2500	3000
-2 m/s	-0.214	-0.174	-0.136	-0.1	-0.065	-0.032	-0.001
-1 m/s	-0.102	-0.068	-0.036	-0.006	0.023	0.051	0.077
0 m/s	0	0.028	0.054	0.08	0.104	0.126	0.148
1 m/s	0.092	0.115	0.136	0.157	0.176	0.195	0.213
2 m/s	0.175	0.192	0.21	0.226	0.241	0.256	0.27

Table 4: Jumping length corrections for women (m).

Wind w(m/s)	Altitude H(m)						
	0	500	1000	1500	2000	2500	3000
-2 m/s	-0.194	-0.16	-0.128	-0.098	-0.069	-0.041	-0.015
-1 m/s	-0.092	-0.064	-0.037	-0.012	0.012	0.035	0.057
0 m/s	0	0.023	0.044	0.065	0.084	0.103	0.121
1 m/s	0.082	0.1	0.118	0.134	0.149	0.164	0.178
2 m/s	0.155	0.169	0.182	0.195	0.207	0.218	0.229

The analysis of these tables shows that the maximal allowable tail-wind 2m/s at sea level will improve the distance of a jump of the same athlete by 14-18 cm. Altitude also will improve a performance on a still day by 4-5 cm for each 1000 m.

According to our calculations a 2 m/s tail-wind at 2250 m altitude (Mexico City's air density) will improve a performance by 24-27 cm for men and by 20-23 cm for women. Our calculations show that the increase of Beamon's result caused by atmospheric conditions could be 29-30 cm which is in good agreement with 31 cm predicted by Ward-Smith [5].

Besides altitude there is another factor affecting the air density - the temperature. All our corrections were calculated for the temperature T=15 °C which corresponds to sea level standard temperature 288.15 K. There is also a rough approximation: a 5 °C temperature increase is approximately equivalent to a 150 m increase of altitude.

It appears that wind and altitude have a significant effect mainly because of changes in take-off values. The faster the athlete runs, the greater the horizontal velocity at the instant he/she touches the take-off board and the greater the take-off velocity. The effects of wind and altitude in the aerial phase can change the distance by no more than 5-6 cm.

4. Conclusions

In this paper we show how wind and altitude can affect the performance of long-jumpers. Besides predictions, diagrams and tables, we also provide a long jump computer simulator for modeling the trajectory of the sportsman's center of gravity. Our program is supposed to help coaches and sportsmen estimate the contribution of atmospheric effects into the final result of a long jump and find out whether the sportsman takes full advantage of his potential or not.

5. References

1. Linthorne NP, Guzman MS, Bridgett LA. Optimum take-off angle in the long jump. Journal of Sports Sciences. 2005; 23:703-712.
2. Mureika JR. The Legality of Wind and Altitude Assisted Performances in the Sprints. New Studies in Athletics. 2000; 15(3/4):53-60.
3. Mureika JR. A Realistic Quasi-Physical Model of the 100 Metre Dash. Can. J Phys. 2001; 79:697-713.
4. Neville de Mestre. A mathematical analysis of wind effects on a long-jumper. J. Austral. Math. Soc. Ser. B. 1991; 33:65-76.
5. Ward-Smith AJ. Altitude and wind effects on long jump performance with particular reference to the world record established by Bob Beamon. Sports Sci. 1986; 4:89-99.