Complex analysis of the long jump taking into consideration atmospheric conditions

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Abstract
It is well known that wind and altitude can affect the performance of long-jumpers. In this paper we show that according to our model the maximal allowable tail-wind 2m/s at sea level will improve the distance of a jump of the same athlete by 14-18 cm. Altitude also will improve the performance on a still day by 4-5 cm for each 1000 m. It can be also calculated that a 5 °C increase of temperature is equivalent to an increase of altitude by 150 m. It appears that wind and altitude have a significant effect mainly because of changes in the take-off values. The faster the athlete runs, the greater the take-off velocity. We also provide a long jump computer simulator for modeling the trajectory of the sportsman’s center of gravity. Our program is supposed to help coaches and sportsmen estimate the contribution of atmospheric effects into the final result and take full advantage of the sportsman’s potential.

Keywords: Long jump, take-off, wind, altitude

1. Introduction
According to IAAF (International Association of Athletics Federations) regulations, sprint and jump performances for which the measured wind-speed exceeds +2.0 m/s are deemed illegal and cannot be ratified for record purposes (IAAF 1998). The average wind component parallel to the track is measured near the jumping pit during an interval encompassing the run-up and the jump. If this measurement exceeds 2 m/s, no record-breaking jump is recognized. Similarly, performances which are achieved at altitudes exceeding 1000 meters above sea level are noted as “altitude-assisted”, but unlike their wind-aided counterparts, these can and have qualified for record status [2]. Indeed, the 1968 Olympics saw amazing World Records set in the men's 100 m, 200 m, and Long Jump, thanks in part to the lofty 2250-meter elevation of Mexico City. In making his record jump, Beamon enjoyed a number of advantageous environmental factors. Mexico City’s air had less resistance than air would have at sea level. This allows runners to run faster and jumpers to jump farther. In addition to Beamon's record, world records were broken in most of the sprinting and jumping events at the 1968 Olympic Games. Beamon also benefited from a trailing wind of 2 meters per second on his jump, the maximum allowable for record purposes. It has been estimated that the trail wind and altitude may have improved Beamon's long jump distance by 31 cm (12.2 inches) [5]. During the same hour Lee Evans set the world record for 400 meters that lasted for almost 20 years. Undoubtedly, wind and altitude affect the performance of sprinters and long-jumpers. The International Athletics Union acknowledges this by imposing a special rule relating to wind during long-jump performances. In this paper we investigate how the atmospheric factors such as wind, altitude and temperature will affect the sportsman during long jump.

2. Materials and methods
2.1 Long Jump Model
To facilitate the study of long jumps, it has been proposed to split the total distance jumped into partial distances, and then to identify the determining factors for each. For the long jump, Linthorn et al. [1] classifies the following partial distances as shown in Figure 1.
L₀: Take-off distance: the horizontal distance between the anterior edge of the take-off board and the vertical projection of the center of gravity (CG) at the instant of take-off.
L₁: Flight distance: the horizontal distance between the CG while the athlete is free in the air.
L₂: Landing distance: the horizontal distance between the vertical projection of the center of gravity at the instant the heels touch the sand and the mark from where the jump will be measured.

The distance L₁ represents more than 85% of the total distance of a jump and thus has the highest relationship with the final result. We can say that L₁, and thus performance in the horizontal jumping events, is determined by the same four factors affecting movement of all projectiles: take-off height, angle and velocity, and air resistance.

In this paper the usual assumption is made that SCₐ and SCᵢ are each some average constant for the duration of each long jump. The value for SCₐ has been estimated as 0.36 for a sportsman weighing 70 kg from measured values on sprinters, cyclists and speed-skaters quoted in Ward-Smith [5]. To obtain some idea of the effect of lift, a representative value 0.04 is chosen for SCᵢ.

2.3 Long Jump Modeling with Wind

When a wind \( \overrightarrow{W} \) is blowing, the air speed of any projectile is given by

\[
\overrightarrow{v}^* = \overrightarrow{v} - \overrightarrow{W}
\]

The drag and lift effects will depend on \( \overrightarrow{V}^* \), and only on \( \overrightarrow{V} \) in the absence of wind. Therefore, with the addition of wind, the basic equation (1) for the projectile part of the long-jumper's motion becomes [4]

\[
m \frac{d^2 \vec{r}}{dt^2} = -mg \hat{k} - \frac{1}{2} \rho_S C_D |\vec{v}|^2 \hat{\vec{r}} + \frac{1}{2} \rho_S C_L |\vec{v}|^2 \hat{\vec{n}}
\]

where \( \vec{r} \) is a unit vector in the direction of \( \vec{v}^* \) and \( \vec{n} \) is a unit vector perpendicular to \( \vec{r} \) and lying in the vertical plane.

It seems take-off velocity is the most important factor affecting L₁ and it has a very high relationship with the velocity at the touchdown of the take-off foot at take-off, which in turn is dependent on the approach velocity. In other words, the faster the athlete runs, the greater the horizontal velocity at the instant he/she touches the take-off board and the greater the take-off velocity.

2.4 Take-off Velocity Calculation

Reference [3] gives a simple formula which can be used to correct 100-meter sprint times according to both wind and altitude conditions,

\[
l_{0,0} \equiv t_{w,H} [1.03-0.03\exp(-0.000125 \cdot H)](1-w_{w,H}/100)^2,
\]

where \( t_{w,H} \) (s) is the recorded race time run with wind \( w \) (m/s) and at altitude \( H \) (m), while the time \( l_{0,0} \) is the adjusted time as if it were run at sea level in calm conditions.

This formula allows us to make a rough approximation of how the take-off velocity of the athlete changes under the influence of wind and altitude. We suppose that the take-off velocity \( V_{t,0,H} \) is proportional to the run-off velocity \( V_{t,run-off}(w,H) \) of the athlete, where the latter can be approximated by the formula \( V_{t,run-off}(w,H) \equiv V_{w,runoff}(w,H)=100 \text{ meter}/t_{w,H} \). Then the sportsman’s take-off speed corresponding to wind \( w \) and altitude \( H \) can be estimated using the following formula:

\[
V_{w,H} \equiv V_{0,0} [l_{0,0}/t_{w,H}]
\]

Using formulas (2)-(5) we can calculate how the result of a long jump depends on wind and altitude. Our long jump distance calculator requires the following input parameters describing the sportsman’s long jump at sea level in still conditions: take-off speed \( V_{t,0,0} \), take-off angle \( \alpha \), take-off height \( h_{1} \), landing height \( h_{2} \) and 100-meter race time \( t_{0,0} \). Then we can calculate the distance of the jump \( L \), its duration \( t \) and the highest position of the sportsman’s center of gravity for the selected values of wind speed \( w \), altitude \( H \) and temperature \( T \). The values of main parameters for elite male

![Diagram of a long jump showing contributions to the official distance.](image)
and female athletes have been taken from reliable sources \cite{1,3} and are shown in Table 1.

Table 1: Values of main parameters.

<table>
<thead>
<tr>
<th>#</th>
<th>Parameter Name</th>
<th>Parameter value</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Take-off speed $V_{0,0}$ (m/s)</td>
<td>9.46</td>
<td>8.35</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Take-off angle $\alpha$ (degree)</td>
<td>22</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Sportsman’s mass $m$ (kg)</td>
<td>80</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Take-off height $h_1$ (m)</td>
<td>1.3</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Take-off distance $L_0$ (m)</td>
<td>0.5</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Landing height $h_2$ (m)</td>
<td>0.65</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Landing distance $L_2$ (m)</td>
<td>0.4</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>100-meter sprint time $t_{100}$ (s)</td>
<td>10</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Drag coefficient $C_D$</td>
<td>0.6599</td>
<td>0.6599</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Lift coefficient $C_L$</td>
<td>0.0733</td>
<td>0.0733</td>
<td></td>
</tr>
</tbody>
</table>

3. Results & Discussion

3.1 Calculations in the still air at sea level

First of all, we calculated how the range of a jump launched at $\alpha$ depends on the starting angle $\alpha$ and the starting velocity of the jump $V_0$. $V_0$ was between 6 m/s and 10 m/s, while $\alpha$ was changing from $15^\circ$ to $25^\circ$. Other parameters: $m$, $h_1$, $h_2$, $L_0$ and $L_2$ have been taken from Table 1 as they are given for male athletes.

The results of calculations are shown in Fig. 2, 3. We see that the increase of $\alpha$ from $15^\circ$ to $25^\circ$ increases the distance by 0.7-2.0 m depending on the value of $V_0$ (Fig. 2). The increase of $V_0$ from 6 m/s to 10 m/s increases the distance by 2.6-5.0 m in an almost linear manner depending on the value of $\alpha$ (Fig. 3).

As you can see, there are a lot of variables that play a role in generating the furthest long jump possible. However, it seems that horizontal velocity is the most important factor in determining overall distance because it directly affects one's flight trajectory and total distance of the jump.

In general, the result of a long jump may be described by the following formula

$$L(V_0, \alpha, m, h_1, h_2, L_0, L_2) = L(V_0, \alpha, m^*, h_1^*, h_2^*, L_0^*, L_2^*) + \Delta L_0 + \Delta h \cdot \text{ctg}(\alpha) + \Delta L_2,$$

where $L^* = L(V_0, \alpha, m^*, h_1^*, h_2^*, L_0^*, L_2^*)$ is the distance corresponding to the elite male athlete (m, $h_1$, $h_2$, $L_0$ and $L_2$ have been taken from Table 1), jumping with the starting parameters $V_0$ and $\alpha$, $\Delta h \cdot \text{ctg}(\alpha) = (h_1 - h_1^* + h_2 - h_2^*) \cdot \text{ctg}(\alpha) - $ is the change of $L$ caused by changes in the take-off height $h_1$ and landing height $h_2$ compared to their values from Table 1 (we assume that during the landing the sportsman’s center of gravity moves along a straight line), $\Delta L_0 = L_0 - L_0^*$ and $\Delta L_2 = L_2 - L_2^*$ are the corrections for $L_0$ and $L_2$.

3.1 Calculations considering wind and altitude

Formulas (2)-(5) from the previous section allow us to calculate long jump results for both males and females and evaluate how the atmospheric conditions affect the final result. The values of main parameters for elite male and female athletes have been taken from Table 1.

Using formula (4) we calculated values of coefficients $(t_0, d, H)$ for different $t_0$, $H$ and $w$ (see Table 2). These coefficients show how a 100-meter sprint time of the sportsman changes under the influence of wind $w$ and altitude $H$. Coefficients have been calculated for four values of $t_0$ between 10 s, 11 s, 12 s and 13 s, altitude $H$ takes six values - 0 m, 500 m, 1000 m, 1500 m, 2000 m and 2500 m, and wind is represented by five values - 2 m/s, -1 m/s, 0 m/s, 1 m/s and 2 m/s. For $t_0$, $w$ and $H$ falling between these values we can use linear approximation to calculate $(t_0, d, H)$. 

Fig 2: The dependencies of $L$ on the starting velocity $V_0$ for different values of $\alpha$.

Fig 3: The dependencies of $L$ on the starting angle $\alpha$ for different values of $V_0$. 

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In Tables 3 and 4, you can see jumping length corrections calculated for male (Table 3) and female (Table 4) elite athletes for different altitudes and different wind speeds.

### Table 3: Jumping length corrections for men (m).

<table>
<thead>
<tr>
<th>Wind w(m/s)</th>
<th>0</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
<th>2500</th>
<th>3000</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2 m/s</td>
<td>-0.214</td>
<td>-0.174</td>
<td>-0.136</td>
<td>-0.1</td>
<td>-0.065</td>
<td>-0.032</td>
<td>-0.001</td>
</tr>
<tr>
<td>-1 m/s</td>
<td>-0.102</td>
<td>-0.068</td>
<td>-0.036</td>
<td>-0.006</td>
<td>0.023</td>
<td>0.051</td>
<td>0.077</td>
</tr>
<tr>
<td>0 m/s</td>
<td>0</td>
<td>0.028</td>
<td>0.054</td>
<td>0.08</td>
<td>0.104</td>
<td>0.126</td>
<td>0.148</td>
</tr>
<tr>
<td>1 m/s</td>
<td>0.092</td>
<td>0.115</td>
<td>0.136</td>
<td>0.157</td>
<td>0.176</td>
<td>0.195</td>
<td>0.213</td>
</tr>
<tr>
<td>2 m/s</td>
<td>0.175</td>
<td>0.192</td>
<td>0.21</td>
<td>0.226</td>
<td>0.241</td>
<td>0.256</td>
<td>0.27</td>
</tr>
</tbody>
</table>

### Table 4: Jumping length corrections for women (m).

<table>
<thead>
<tr>
<th>Wind w(m/s)</th>
<th>0</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
<th>2500</th>
<th>3000</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2 m/s</td>
<td>-0.194</td>
<td>-0.16</td>
<td>-0.128</td>
<td>-0.098</td>
<td>-0.069</td>
<td>-0.041</td>
<td>-0.015</td>
</tr>
<tr>
<td>-1 m/s</td>
<td>-0.092</td>
<td>-0.064</td>
<td>-0.037</td>
<td>-0.012</td>
<td>0.012</td>
<td>0.035</td>
<td>0.057</td>
</tr>
<tr>
<td>0 m/s</td>
<td>0</td>
<td>0.023</td>
<td>0.044</td>
<td>0.065</td>
<td>0.084</td>
<td>0.103</td>
<td>0.121</td>
</tr>
<tr>
<td>1 m/s</td>
<td>0.082</td>
<td>0.1</td>
<td>0.118</td>
<td>0.134</td>
<td>0.149</td>
<td>0.164</td>
<td>0.178</td>
</tr>
<tr>
<td>2 m/s</td>
<td>0.155</td>
<td>0.169</td>
<td>0.182</td>
<td>0.195</td>
<td>0.207</td>
<td>0.218</td>
<td>0.229</td>
</tr>
</tbody>
</table>

The analysis of these tables shows that the maximal allowable tail-wind 2 m/s at sea level will improve the distance of a jump of the same athlete by 14-18 cm. Altitude also will improve a performance on a still day by 4-5 cm for each 1000 m. According to our calculations a 2 m/s tail-wind at 2250 m altitude (Mexico City’s air density) will improve a performance by 24-27 cm for men and by 20-23 cm for women. Our calculations show that the increase of Beamon’s result caused by atmospheric conditions could be 29-30 cm which is in good agreement with 31 cm predicted by Ward-Smith [5].

Besides altitude there is another factor affecting the air density - the temperature. All our corrections were calculated for the temperature T=15 °C which corresponds to sea level standard temperature 288.15 K. There is also a rough approximation: a 5 °C temperature increase is approximately equivalent to a 150 m increase of altitude. It appears that wind and altitude have a significant effect mainly because of changes in take-off values. The faster the athlete runs, the greater the horizontal velocity at the instant he/she touches the take-off board and the greater the take-off velocity. The effects of wind and altitude in the aerial phase can change the distance by no more than 5-6 cm.

### 4. Conclusions

In this paper we show how wind and altitude can affect the performance of long-jumpers. Besides predictions, diagrams and tables, we also provide a long jump computer simulator for modeling the trajectory of the sportsman’s center of gravity. Our program is supposed to help coaches and sportsmen estimate the contribution of atmospheric effects into the final result of a long jump and find out whether the sportsman takes full advantage of his potential or not.

### 5. References